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Dear Students,

This Institute is dedicated to cater the needs of students preparing for Indian Statistical Service. We publish videos on Youtube channel for student help.

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Vital Statistics ⇒ Vital Statistics (जीवनाधार सांख्यिकी) is defined as that branch of biometry which deals with data and the Laws of Human mortality (मृत्यु दर), morbidity (रोगों की संख्या) and Demography (जनपद विज्ञान)

In Vital Statistics theory we Analysis the data related to Vital Events (births, deaths, marriage, divorce, separation, adoption, etc)

Uses of Vital-Statistics ⇒ Vital statistics are being extensively used in almost all the areas of Human activity

(i) Study of popn Trend ⇒ The study of births (fertility) and deaths (mortality) gives us an idea of the popn trend of any Region, Community or Country

If Birth Rate > Death Rate → Increasing trend

< → Decreasing "

(II) Use in Public Administration ⇒
The study of popⁿ movement i.e. popⁿ estimation, popⁿ projections, birth and death rates according to age and Sex distributions provides to any administration with fundamental tools which are Important for the overall planning of Economic and Social development programmes.

(III) Use In Medical Science ⇒ Mortality and fertility statistics also provide guide spots for use by the Researchers in the medical and Pharmaceutical profession.

(IV) Use to operating Agencies ⇒ The facts and figures related to births, deaths, and marriages are of extreme Importance to Various official agencies for a Variety of administrative purposes.

· Mortality statistics serve as a guide to the Health authorities for Sanitary (स्वास्थ्य) Improvements, Improved medical facilities.

(V) Use in Insurance Sector ⇒

The whole of actuarial Science, including life Insurance is based on the mortality or life tables. the vital records concerning all possible factors contributing to deaths in various ages are compulsory tool in numerous life Insurance Schemes.

Methods of obtaining Vital statistics ⇒ (data) ⇒

(i) Registration Method ⇒

The most important source of obtaining vital statistics data is the registration method which consists in continuous and permanent recording of vital events pertaining to births, deaths, marriages, migration, etc.

These data, in addition to their statistical utility, also have their value as legal documents. Many countries require compulsory registration of births and deaths under the law.

drawback of Registration Method ⇒

In India, in rural areas there is no laws which makes the registration of the vital events (births, deaths) and reporting of epidemics compulsory. So consequently, a number of births are likely to remain unregistered especially in Rural areas. thus in India, the statistics of births suffer from the error of underestimation.

(ii) Census Method ⇒ Almost in all the countries, all over the world, pop^n census is conducted at regular intervals of time, usually ten years.

Census consists of complete enumeration of the pop^n of the particular area under study and collecting information from individuals regarding age, sex, marital status, occupation, and other economic and social characteristic.

Drawback of the census ⇒

The main drawback of the census method is that it provides vital statistics only for the census year and fails to give any information about the vital events in the intercensal period.

Measurement of Mortality ⇒

The following are the principal Rates used in measuring mortality.

(i) Crude Death Rate (C.D.R) ⇒
(कच्चा)(अधूरा)

This is the Simplest of all the indices of mortality and is defined as the number of deaths (from all causes) "per" K persons in the pop^n of any given Community or Region during a given period.

In particularly, the Annual Crude Death rate C.D.R defined as

$$m = \frac{\text{Annual deaths}}{\text{Anual Mean pop}^n} \times K \quad \{ K = 1000 \text{ in general.}$$

The Crude death rate for any period gives the Rate at which the pop^n is depleted (खाली) through deaths over the Course of the period.

Merits of C.D.R ⇒

(i) it is Simple to understand and calculate

(ii) C.D.R is perhaps the most widely used of any vital statistics rates. As an Index of mortality, it is used in numerous demographic and public Health problems.

(iii) C.D.R is a probability rate giving the prob^t that a person belonging to the giving pop^n will die in the given period.

Demerits ⇒ Most serious drawback of C.D.R is that it completely ignores the age, and Sex distribution. but experience shows that mortality is different in different segments of the pop^n.

C.D.R is not suitable for comparing the mortality in Two places or same place in Two different periods.

NOTE - we can compute the C.D.R for males and females separately.

$$\text{C.D.R for males} = \frac{\text{Male death}}{\text{Male pop}^n} \times 1000 = \frac{m_D}{m_P} \times 1000$$

$$\text{C.D.R for females} = \frac{\text{Female death}}{\text{Female pop}^n} \times 1000 = \frac{f_D}{f_P} \times 1000$$

NOTE - C.D.R usually lies between 8 and 30 per thousand and female C.D.R is generally less than male C.D.R

(ii) Specific Death Rates (S.D.R) ⇒
(विशेष)

Since Mortality pattern is different in different Segments of the pop^n i.e age, sex, occupation, social status etc. so we calculate some specific Death Rates for better analysation.

Death rate computed for a particular Section of the pop^n is termed as S.D.R (specific death rate).

$$S.D.R = \frac{\text{Total no of deaths in the specific section of the pop}^n\text{ in the given period}}{\text{Total pop}^n\text{ of the specified section in the same period}} \times K$$

In general $K = 1000$.

Age-Specific Death Rate (Age-S.D.R) –:

${}_nD_x$ = number of deaths in age-group $(x, x+n)$ i.e number of deaths among the persons with age x or more but less than $x+n$ i.e $x \le age < x+n$, in a given region during a given period, t.

${}_nP_x$ = Total pop^n in the age-group $[x, x+n)$

then the Age-specific death rate is given by

$${}_nm_x = \frac{{}_nD_x}{{}_nP_x} \times 1000$$

If $n=1$, we get annual age - S.D.R is

$${}_1m_x = m_x = \frac{D_x}{P_x} \times 1000.$$

NOTE :- To be more specific, the Age - S.D.R for males is given by $${}^m_nm_x = \frac{{}^m_nD_x}{{}^m_nP_x} \times 1000$$

${}^m_nD_x$ = no of death of males pop^n in the age group $[x, x+n)$

${}^m_nP_x$ = no of males in the pop^n in the age group $[x, x+n)$

Similarly :

age - S.D.R for female $= {}^f_nm_x = \frac{{}^f_nD_x}{{}^f_nP_x} \times 1000$

Merits ⇒

(i) The death rates specific to age and sex overcome the drawback of C.D.R, since they are computed by taking into consideration the age and sex composition of the pop^n.

(ii) it also supplies one of the essential components required for computation of Net Reproduction Rate.

Demerits ⇒ (i) However S.D.R's are not of much utility for overall comparison of mortality conditions prevailing (प्रचलित) in two different Regions, say A and B.

For example, it might happen that for certain age-groups the mortality pattern for Region A is greater than that for B. but for other age-groups the case may be opposite.

(ii) In addition to age and sex distribution of the pop^n social, occupational, and topographical factors come into operation. The S.D.R's completely ignore these factors.

Infant Mortality Rate (I.M.R.) ⇒
(शिशु मृत्यु दर)

The Infant mortality rate is defined as the chance of dying of a newly born infant (शिशु) within a year of its life, under the given mortality conditions.

Notations :-

$D_0^z \rightarrow$ number of deaths (Excluding foetal (भ्रूण) death) among the children between the age group 0-1. i.e the number of deaths among the children of age 0 on last birthday (l.b.d.) among the residents of a Region during the calendar year z.

$B_0^z \rightarrow$ Total no of live births reported in the same region within the same calendar year z.

The Infant mortality Rate during the calendar year z, denoted by $\dot{i}_m^z$, is given by

$$I.M.R = \dot{i}_m^z = \frac{D_0^z}{B_0^z}$$

Mortality Table or life Table ⇒

The life Table gives the life history of a Hypothetical group as it is gradually diminished by deaths.

A life table provides answers to the following questions.

(i) How will a group of Infants (शिशुओं) all born at the same time and experiencing unchanging mortality conditions throughout the life time, gradually die out (धीरे धीरे मरना).

(ii) when in the Course of Time (अवधि) all these Infants die, what would be the average Longevity (औसत आयु) per person.

(iii) What is the $prob^t$ that persons of specified age will Survive a specified number of years.

(iv) How many persons, out of selected number of persons living at some initial age. Survive on the average to each attained age.

Notations ⇒

$l_x \rightarrow$ is the number of persons living at any specified age x, in any year out of an assumed number of births, say l_0 usually called the cohort or Radix (मूल) of the life table.

$d_x \rightarrow$ is the number of persons among the l_x persons (attaining a precise age x) who die before reaching the age $(x+1)$.

$$\Rightarrow \quad d_x = l_x - l_{x+1} = -\Delta l_x \qquad \{\Delta l_x = l_{x+1} - l_x$$

${}_nP_x \rightarrow$ is the prob^t that a person aged x survives up to age $x+n$.

$$\Rightarrow \quad {}_nP_x = \frac{l_{x+n}}{l_x} \Rightarrow l_{x+n} = l_x \cdot {}_nP_x$$

If $n=1$, then

$${}_1P_x \equiv P_x = \frac{l_{x+1}}{l_x}$$

which gives the prob^t that a person aged x will survive till his next birthday.

"i.e it must belong to the pop^n of l_{x+1}."

$q_x \equiv {}_1q_x = 1 - P_x \rightarrow$ is the prob^t that a person of exact age x will die within one year

following the attainment of that age.
"i.e the person not belong in the pop^n l_{x+1}."

$$\Rightarrow q_x = \frac{d_x}{l_x} \qquad \{ l_x = l_{x+1} + d_x \}$$

$L_x \rightarrow$ is the number of Persons/years lived in the aggregate (altogether) by the Cohort of l_0 persons between age x and $x+1$.

Thus, if deaths are assumed to be uniformly distributed over the whole year i.e

If we assume that linearity of l_{x+t} $\forall$ $t \in [0,1]$

$$\Rightarrow L_x = \int_0^1 l_{x+t}\, dt \qquad \text{and} \qquad l_{x+t} = l_x - t d_x.$$

$$\Rightarrow L_x = \int_0^1 (l_x - t d_x)\, dt = \left[l_x \cdot t - d_x \cdot \frac{t^2}{2} \right]_0^1 = l_x - \frac{d_x}{2}$$

$$= l_x - \frac{1}{2}(l_x - l_{x+1}) = \frac{1}{2}(l_x + l_{x+1})$$

$$\Rightarrow L_x = l_{x+\frac{1}{2}} \qquad \left\{ \begin{array}{l} l_{x+t} = l_x - t d_x \\ \Rightarrow l_{x+\frac{1}{2}} = l_x - \frac{1}{2} d_x = L_x. \end{array} \right.$$

$T_x \rightarrow$ is the Total number of years lived by the cohort l_0 after attaining the age x i.e

T_x is the total future life time of the l_x persons who reach age x.

$$\Rightarrow T_x = L_x + L_{x+1} + L_{x+2} + \cdots\cdots$$

If w is the highest age at which any survivors are recorded in the mortality Table, i.e. $l_w = 0$.

then $T_x = \sum_{J=x}^{w-1} L_J = \sum_{i=0}^{w-1-x} L_{x+i}$

and $T_x = L_x + T_{x+1}$.

Thm $\Rightarrow$ ${}_nP_x = P_x \cdot P_{x+1} \cdots P_{x+n-1}$.

proof $\Rightarrow$ ${}_nP_x = \dfrac{l_{x+n}}{l_x} = \dfrac{l_{x+1}}{l_x} \cdot \dfrac{l_{x+2}}{l_{x+1}} \cdots\cdots \dfrac{l_{x+n}}{l_{x+n-1}}$.

$\Rightarrow {}_nP_x = P_x \cdot P_{x+1} \cdots P_{x+n-1}$

Th^m $\Rightarrow$ ${}_nq_x = \dfrac{d_{x+n-1}}{l_x}$.

Proof:- ${}_nq_x$ = Prob^t that a person aged x dies in the n^{th} year after attaining the age x.

$= P$[A Person aged x survives till age $(x+n-1)$ but dies in age period $(x+n-1, x+n)$]

$= P$[A Person aged x survives for $(n-1)$ years] $\times$ P[A Person aged $(x+n-1)$ dies within one year]

$$= \frac{l_{x+n-1}}{l_x} \cdot \frac{d_{x+n-1}}{l_{x+n-1}} = \frac{d_{x+n-1}}{l_x}$$

NOTE - $${}_{n}p_x - {}_{n+1}p_x = \frac{l_{x+n}}{l_x} - \frac{l_{x+n+1}}{l_x} = \frac{d_{x+n}}{l_x} = {}_{n+1}q_x$$

Th^m ⇒ If ω is the last age at which l_x vanishes

ie $l_\omega = 0$ then $l_x = \sum_{i=x}^{\omega-1} d_i$.

Proof ⇒ $$\sum_{i=x}^{\omega-1} d_x = d_x + d_{x+1} + \cdots + d_{\omega-1}$$

$$= (l_x - l_{x+1}) + (l_{x+1} - l_{x+2}) + \cdots + (l_{\omega-1} - l_\omega) = l_x$$

Th^m ⇒ $$T_x = \frac{1}{2} l_x + l_{x+1} + l_{x+2} + \cdots\cdots$$

Proof ⇒ $$T_x = \sum_{t=0}^{\infty} L_{x+t} = \sum_{t=0}^{\infty} \frac{1}{2}\left(l_{x+t} + l_{x+t+1}\right)$$

$$= \frac{1}{2} l_x + \frac{1}{2} l_{x+1} + \frac{1}{2} l_{x+1} + \frac{1}{2} l_{x+2} + \cdots\cdots$$

$$= \frac{1}{2} l_x + \sum_{t=1}^{\infty} l_{x+t} .$$

Expectation of life ⇒

(i) The Curate Expectation of life ⇒
(Complete year Expectation)

⇒ The curate expectation of life, usually denoted by C_x gives the average number of complete years of life lived by the cohort l_0 after age x. i.e. l_x persons.

(ii) Complete Expectation of life ⇒ usually denoted by $\overset{\circ}{C}_x$ measures the average number of years a person of given age x can be expected to live under the prevailing mortality conditions.

It gives the number of years of life entirely completed and includes the fraction of the year survived in the year in which death occurs, which on the average can be taken to be 1/2 year. so

$$\boxed{\overset{\circ}{e}_x = e_x + \frac{1}{2}}$$

NOTE :-

Since Total number of years lived by l_x persons of age x is given by

$$T_x = \int_0^\infty l_{x+t}\, dt$$

$$\Rightarrow \quad \overset{\circ}{e}_x = \frac{T_x}{l_x}$$

and $\overset{\circ}{e}_0 \Rightarrow$ is the expectation of life at age 0, is the average age at death of a person belonging to a given community.

Th^m ⇒ Show that $c_x = \left(\sum_{n=1}^{\infty} l_{x+n}\right)/l_x$.

Proof:- Since d_x is number of persons dying in the first year without completing one year of life at age x.

⇒ Total no of years lived by d_x individuals $= 0 \cdot d_x = 0$

Total no of years lived by d_{x+1} individuals $= 1 \cdot d_{x+1} = d_{x+1}$.

" " " " " " d_{x+i} " $= i\, d_{x+i}$.

$$\Rightarrow c_x = \frac{\sum_{i=0}^{\infty} i \cdot d_{x+i}}{l_x} = \frac{1}{l_x}\left[d_{x+1} + 2d_{x+2} + 3d_{x+3} + \cdots\right]$$

$$= \frac{1}{l_x}\left[(l_{x+1} - l_{x+2}) + 2(l_{x+2} - l_{x+3}) + \cdots\right]$$

$$= \frac{1}{l_x}\left[l_{x+1} + l_{x+2} + l_{x+3} + \cdots\right] = \frac{1}{l_x}\sum_{n=1}^{\infty} l_{x+n}$$

NOTE:- Since $l_x \cdot c_x = l_{x+1} + l_{x+2} + l_{x+3} + \cdots$

and $l_{x+1} \cdot c_{x+1} = l_{x+2} + l_{x+3} + \cdots$

$$\Rightarrow l_x c_x - l_{x+1} \cdot c_{x+1} = l_{x+1}$$

$$\Rightarrow \frac{l_{x+1}}{l_x} = \frac{c_x}{1 + c_{x+1}} \Rightarrow p_x = \frac{c_x}{1 + c_{x+1}}$$

Similarly $q_x = 1 - p_x \Rightarrow$ $$q_x = \frac{1 - (c_x - c_{x+1})}{1 + c_{x+1}}.$$

Stationary pop^n ⇒ A Stationary pop^n is a Stable pop^n in which the Intrinsic growth Rate is zero. i.e. difference of birth Rate and death rate is zero. Hence none of the pop^n Variables in a stationary pop^n change over Time. i.e The annual number of births, the annual no of deaths, pop^n size, the size of a certain age group, the size of certain Sex are Constant over Time. i.e the pop^n will be of the same size from year to year and will have the same age-distribution So that the number of persons between the age x and $(x+1)$, L_x, will always be same.

Stable pop^n ⇒ A pop^n is said to be stable if

(i) It has a fixed age and Sex distribution

(ii) Constant mortality and fertility Rates are experienced at each age.

(iii) The pop^n is closed (बंद) to emigration or Immigration i.e

In a stable pop^n, Mortality and fertility Rates are Constant but need not be equal.

so for a stable pop^n the overall Rates of births and deaths remains Constant and Consequently such a pop^n increases or decreases at a Constant Rate.

NOTE: Stationary pop^n $\Rightarrow$ stable
$\not\Leftarrow$

Lotka and Dublin's model for 'Stable pop^n' ⇒
" In this model we estimate growth rate of a stable pop^n."

Assumptions :- Lotka and Dublin's stable pop^n Analysis is based on the assumptions.

(i) The birth Rates are Independent of time, t.

(ii) " Death " " " "

(iii) The Age distribution between the ages x to $(x+\delta x)$ is Independent of t.

(iv) The pop^n is closed to migration i.e Emigration and Immigration is not allowed.

(v) The Analysis is done with respect to female pop^n only.

Notations ⇒

$p(t)$ → Size of the pop^n at Any time t.

$c(x,t)\,\delta x$ → The proportion of pop^n in age Interval $(x, x+\delta x)$ at time t.

$B(t)$ → Total number of births at time t (only femail)

$p(x)$ → prob^t that a female child (born alive) will Survive upto age x under the given mortality conditions.

$i(x)\,\delta x \rightarrow$ prob^t that a woman aged x will give birth to a female child in the age Interval $(x, x+\delta x)$ under the given fertility conditions

$\Rightarrow P(t)\cdot c(x,t)\cdot \delta x$ = pop^n (female only) in the age group x to $(x+\delta x)$ at time t.

$B(t-x)\cdot p(x)\,\delta x$ = A group of persons (female) born $(t-x)$ years ago will survive upto age x or age Interval $(x, x+\delta x)$ at time t.

= Number of persons in the age group $(x, x+\delta x)$ at time t.

$$\Rightarrow \quad P(t)\, c(x,t)\,\delta x = B(t-x)\cdot p(x)\cdot \delta x$$

$$\Rightarrow \quad P(t)\, c(x,t) = B(t-x)\, p(x) \quad \text{—(i)}$$

$$\Rightarrow \int_0^\infty P(t)\, c(x,t)\, i(x)\,dx = \int_0^\infty B(t-x)\, p(x)\, i(x)\,dx \quad \text{—(ii)}$$

Here R.H.S in Eq^n (ii) i.e.

$\int_0^\infty B(t-x)\,p(x)\,i(x)\,dx = \int_0^\infty$[A group of women born $(t-x)$ years ago will survive upto age x and give birth to a female child in the age interval $(x, x+\delta x)$] dx

= a group of women born $(t-x)$ years ago would replace themselves by future mothers after a period of x year i.e at time t

= Number of births at time t.

= $B(t)$

$$\Rightarrow \quad B(t) = \int_0^\infty B(t-x) \cdot p(x) \cdot i(x)\, dx. \quad \text{— (3)}$$

Eq^n (3) is an Integral Eq^n with Lag x. which is not easily Solable. so, Lotka and Dublin Suggested a trial solution of the form.

$$B(t) = \sum_{n=0}^{\infty} a_n e^{r_n t} \quad \text{— (4)}$$

where $a_0, a_1, a_2, \ldots$ are the sizes of the pop^n. at the beginning of each year under consideration. and

$r_0, r_1, r_2, \ldots$ are the Corresponding growth Rates of pop^n over time.

so by 3 and 4.

$$\sum_{n=0}^{\infty} a_n e^{r_n t} = \int_0^\infty \left\{ \sum_{n=0}^{\infty} a_n e^{r_n (t-x)} \right\} p(x)\, i(x)\, dx$$

$$\Rightarrow \sum_{n=0}^{\infty} a_n e^{r_n t} = \sum_{n=0}^{\infty} a_n e^{r_n t} \int_0^\infty e^{-r_n x} p(x)\, i(x)\, dx.$$

which will be true iff $\int_0^\infty e^{-r_n x} p(x)\, i(x)\, dx = 1$

$\forall\ n = 0, 1, 2, \ldots$

Here $r_0, r_1, r_2, \ldots$ are the Roots of the Eq^n

$$\int_0^\infty e^{-rx} p(x)\, i(x)\, dx = 1 \quad \text{— (5)}$$

Eq^n (5) is known as Lotka's Integral Eq^n.

Lotka's Integral Eq^n can be rewritten as

$$\int_0^\infty e^{-rx}\phi(x)\,dx = 1 \quad \text{— (6)}$$

where $\phi(x) = p(x)\,l(x)$ = Net maternity f^n — (7)
(मातृत्व) child bearing period

Lotka proved that; of all the Infinite number of roots of the Integral Eq^n (5), only one root is real and the remaining all other are complex. Also, the only real root is the most dominant root of Lotka's Integral Eq^n and is supposed to ultimately reflect the changes in the pop^n. i.e It can be regarded as a measure of the growth rate of the pop^n.

let r be the only real root of Eq^n (6).
our aim is to obtain an Estimate of r.
for this, we proceed as follows.

let us write $$\int_0^\infty e^{-rx}\phi(x)\,dx = y \quad \text{—(8)}$$

Differentiating w.r. to r, we get.

$$\frac{dy}{dr} = -\int_0^\infty x\, e^{-rx}\, \phi(x)\, dx$$

$$\Rightarrow \frac{dy}{dr} = -\left[\frac{\int_0^\infty x\cdot e^{-rx}\phi(x)\,dx}{\int_0^\infty e^{-rx}\phi(x)\,dx}\right]\int_0^\infty e^{-rx}\phi(x)\,dx = -A(r)\cdot y \quad \text{—(9)}$$

where $A(r) = \int_0^\infty x\cdot e^{-rx}\phi(x)\,dx \Big/ \int_0^\infty e^{-rx}\phi(x)\,dx$. —(10)

is a function of r.

$\Rightarrow \frac{dy}{y} = -A(r)\,dr$ by eq^n (9)

Integrating both side, we get.

$$\log y = -\int A(r)\,dr + \log c \Rightarrow y = c\cdot e^{-\int A(r)dr} \quad \text{—(11)}$$

$$\Rightarrow 1 = c\cdot e^{-\int A(r)dr} \Rightarrow c = e^{\int A(r)dr} \quad \text{—(12)}$$

let when $r = 0$, $y = c \Rightarrow \log y = \int A(r)\,dr$ —⊛.

$\Rightarrow$ by eq^n (8)

$$y = c = \int_0^\infty \phi(x)\,dx = R_0 = NRR \text{ per woman}$$

NRR $\downarrow$ Net Reproduction Rate.

$$\Rightarrow \log_e y = \log_e R_0 \quad \text{—(13)}$$

Now Since $e^z = \sum_{J=0}^\infty \frac{z^J}{J!} \Rightarrow e^{-rx} = \sum_{J=0}^\infty \frac{(-rx)^J}{J!}$

$$\Rightarrow A(r) = \int_0^\infty x \sum_{J=0}^\infty (-r)^J\cdot x^J/J!\cdot\phi(x)\,dx \Bigg/ \int_0^\infty \sum_{J=0}^\infty \frac{(-r)^J\cdot x^J}{J!}\cdot\phi(x)\,dx$$

$$\Rightarrow A(\pi) = \frac{\sum_{J=0}^{\infty}\left[\frac{(-\pi)^J}{J!}\int_0^{\infty} x^{J+1}\,\phi(x)\,dx\right]}{\sum_{J=0}^{\infty}\left[\frac{(-\pi)^J}{J!}\int_0^{\infty} x^{J}\,\phi(x)\,dx\right]} = \frac{\sum_{J=0}^{\infty}\frac{(-\pi)^J}{J!}R_{J+1}}{\sum_{J=0}^{\infty}\frac{(-\pi)^J}{J!}R_J}$$

where $R_J = \int_0^{\infty} x^J\,\phi(x)\,dx$

$$\Rightarrow A(\pi) = \frac{R_1\left[1 - \pi\, R_2/R_1 + \frac{\pi^2}{2!} R_3/R_1 - \frac{\pi^3}{3!} R_4/R_1 + \cdots\right]}{R_0\left[1 - \pi\, R_1/R_0 + \frac{\pi^2}{2!} R_2/R_0 - \frac{\pi^3}{3!} R_3/R_0 + \cdots\right]}$$

$$= \frac{R_1}{R_0}\left[1 + \left(\frac{R_1}{R_0} - \frac{R_2}{R_1}\right)\pi + \frac{1}{2}\left\{\frac{R_3}{R_1} - \frac{3R_2}{R_0} + 2\left(\frac{R_1}{R_0}\right)^2\right\}\pi^2 + \cdots\right]$$

{by Long division}

$$= \frac{R_1}{R_0} + \left[\left(\frac{R_1}{R_0}\right)^2 - \frac{R_2}{R_0}\right]\pi + \frac{1}{2}\left[\frac{R_3}{R_0} - \frac{3R_1R_2}{R_0} + 2\left(\frac{R_1}{R_0}\right)^3\right]\pi^2 + \cdots$$

$$= \alpha + \beta\pi + \gamma\pi^2 + \cdots \qquad \text{— (14)}$$

where $\alpha = \frac{R_1}{R_0}$; $\beta = \left(\frac{R_1}{R_0}\right)^2 - \frac{R_2}{R_0}$; $\gamma = \frac{1}{2}\left[\frac{R_3}{R_0} - \frac{3R_1R_2}{R_0} + 2\left(\frac{R_1}{R_0}\right)^3\right]$

Neglecting terms involving π^2 and Higher power of π, we have

$$A(\pi) \cong \alpha + \beta\pi \Rightarrow \int A(\pi)\,d\pi \cong \alpha\pi + \frac{\beta\pi^2}{2}$$

$$\Rightarrow \log_e R_0 \cong \alpha\pi + \frac{\beta\pi^2}{2} \qquad \text{by eq}^n \ * \text{ and (13)}$$

$$\Rightarrow \beta\pi^2 + 2\alpha\pi - 2\log_e R_0 = 0$$

$$\Rightarrow \pi = \frac{-2\alpha \pm \sqrt{4\alpha^2 + 8\beta\log_e R_0}}{2\beta} = \frac{-\alpha \pm \sqrt{\alpha^2 + 2\beta\log_e R_0}}{\beta} \qquad \text{— (15)}$$

$$\Rightarrow x = \frac{-\frac{R_1}{R_0} \pm \sqrt{\left(\frac{R_1}{R_0}\right)^2 + 2\left\{\left(\frac{R_1}{R_0}\right)^2 - \frac{R_2}{R_0}\right\}\log_e R_0}}{\left(\frac{R_1}{R_0}\right)^2 - \frac{R_2}{R_0}} \quad \text{--- (16)}$$

where R_0, R_1 and R_2 are estimated as.

$$\hat{R}_0 = NRR = \sum_x p(x)\, i(x) = \sum_x \phi(x)$$

$$\hat{R}_1 = \frac{\sum_x x\, p(x)\, i(x)}{\sum_x p(x)\, i(x)} = \frac{\sum x\,\phi(x)}{\sum \phi(x)} = \text{Mean age of child bearing period.}$$

$$\hat{R}_2 = \frac{\sum x^2\, p(x)\, i(x)}{\sum p(x).\, i(x)} = \frac{\sum x^2 \phi(x)}{\sum \phi(x)}$$

put these values of $\hat{R}_0$, $\hat{R}_1$ and $\hat{R}_2$ in Eq^n (16) we get Two values of x, one is positive and other is negative.

we choose x positive if birth Rate > death Rate

" " negative " " " < " "

Mean and Variance of Net maternity function ⇒

$\phi(x) = p(x) \cdot l(x)$ is the net maternity f^n or child bearing period and the prob^t density f^n of Net maternity f^n x is given by

$$\psi(x) = \frac{\phi(x)}{\int_0^\infty \phi(x)\,dx}$$

Since $R_K = \int_0^\infty x^K \phi(x)\,dx$, then

Mean Age of child bearing period i.e $E(x)$ is

$$E(x) = \int_0^\infty x \cdot \psi(x)\,dx = \frac{\int_0^\infty x\,\phi(x)\,dx}{\int_0^\infty \phi(x)\,dx} = \frac{R_1}{R_0} = \alpha$$

$$V(x) = E(x^2) - \{E(x)\}^2 = \frac{\int_0^\infty x^2 \phi(x)\,dx}{\int_0^\infty \phi(x)\,dx} - \left(\frac{R_1}{R_0}\right)^2 = \frac{R_2}{R_0} - \left(\frac{R_1}{R_0}\right)^2 = \beta$$

Central Mortality Rate ⇒ The central mortality or death Rate is the prob^{t} that a person whose exact age is "not Known" but lies in between x and $(x+1)$ will die within the year. it is denoted by m_x

$$m_x = \frac{\text{Number of deaths within age-Interval } (x, x+1)}{\text{Average } l_x \text{ of the cohort in } (x, x+1)}$$

$$\Rightarrow m_x = \frac{d_x}{L_x} = \frac{d_x}{l_x - \frac{1}{2} d_x} = \frac{2\, d_x/l_x}{2 - d_x/l_x} = \frac{2 q_x}{2 - q_x}$$

$$\Rightarrow q_x = \frac{2 m_x}{2 + m_x}.$$

Force of Mortality ⇒ So far we have confined (सीमित) ourselves to the values of l_x for integral values of x. But since deaths occur at all ages and at every fraction of time of the year, l_x is a continuous f^n of x. So

At any age x, the rate of decrease in l_x is given by

$$\lim_{t \to 0} \frac{l_x - l_{x+t}}{t} = -\lim_{t \to 0} \frac{l_{x+t} - l_x}{t} = -\frac{dl_x}{dx}$$

The force of mortality at age x is defined as the ratio of instantaneous rate of decrease in l_x to the value of l_x, denoted as μ_x.

$$\mu_x = -\frac{1}{l_x} \cdot \frac{dl_x}{dx} = -\frac{d}{dx}(\log l_x)$$

It gives nominal annual rate of mortality i.e. the prob^t of a person of age x exactly dying within the year if the risk of dying is same at every moment of the year as it is during the moment following the attainment of age x.

Th^m $\Rightarrow$ $\mu_{x+\frac{1}{2}} = m_x$

Proof:- Since $L_x = \int_0^1 l_{x+t}\, dt$

$$\Rightarrow \frac{d}{dx} L_x = \frac{d}{dx} \int_0^1 l_{x+t}\, dt = \int_0^1 \frac{d}{dx}(l_{x+t})\, dt$$

$$= \int_0^1 \frac{d}{dt}(l_{x+t})\, dt \qquad \left\{ l_{x+t} \text{ is continuous both in } x \text{ and } t \right.$$

$$= \left[l_{x+t} \right]_0^1 = l_{x+1} - l_x = -d_x$$

$$\Rightarrow \frac{d_x}{L_x} = -\frac{1}{L_x} \cdot \frac{d}{dx}(L_x) \Rightarrow m_x = -\frac{1}{l_{x+\frac{1}{2}}} \cdot \frac{d}{dx}(l_{x+\frac{1}{2}})$$

$$\Rightarrow m_x = \mu_{x+\frac{1}{2}}$$

Assumptions, Description and Construction of Life Tables ⇒

Assumptions -

(I) The cohort l_0 is closed for emigration or Immigration.

(II) Individuals die at each age according to pre-determined schedule which is fixed and does not change.

(III) The cohort originates from some standard number of births, say, 10,000 or 1,00,000. which is called the Radix of the table.

(IV) the deaths are distributed Uniformly over the period $(x, x+1)$ for each x. (except for first few years)

Description of a Life Table -
(विवरण)

A Typical life Table has generally the following columns.

1	2	3	4	5	6	7	8
x	l_x	d_x	q_x	L_x	T_x	e_x^0	e_x

Construction of Life Table ⇒

The complete life table can be constructed if we can compute the quantities q_x $\forall$ $x \geq 0$ and we need only Radix l_0.

The q_x column is thus called the pivotal column of the life table.

Abridged Life Table ⇒ In the complete life table, the age interval is a year throught the table and the life table functions such as l_x, d_x, q_x, m_x etc are given for "all" Integral values of x. i.e (l_x, d_x, q_x etc हर साल के लिये बनेंगे)

on the other hand in Abridged life table the values of these functions i.e l_x, q_x, d_x etc are given.

(i) either for some Integral values of x which are at some distance apart, usually 5 years or 10 years i.e (l_1, l_6, l_{11} .... etc)

(ii) or they are i.e (l_x, d_x, q_x) given for age groups of values of x, usually of width 5 year or 10 year i.e (${}_nd_x$, no of deaths in age Interval $(x, x+n)$)

NOTE - we use Abridged life table when we compare vital events for different age-groups or we compare w.r.t. to different time period.

Construction of Abridged life table ⇒

The principle methods used for the construction of Abridged life table are

(i) Reed-Merrel Method

(ii) Greville's Method

(iii) King's Method

NOTE. Reed-Merrel method and Greville's method is used when x is Integral value which are at some distance apart and King's method is used when we have some age-groups of values of x.

Notations $\Rightarrow$

A typical abridged life table consists of the following columns.

(i) Exact age Intervals x to $(x+n)$ i.e. $x_0, x_0+n, x_0+2n, \ldots$

(ii) l_x, the number of persons out of a cohort of the l_0 persons, living at the beginning of the Interval x to $x+n$.

(iii) ${}_nq_x \rightarrow$ the prob^t of the person dying in the age Interval x to $x+n$ and is given by

$${}_nq_x = 1 - {}_nP_x = 1 - \frac{l_{x+n}}{l_x}$$

(iv) ${}_nd_x \rightarrow$ the number of deaths in the age interval x to $x+n$. and

$${}_nq_x = \frac{{}_nd_x}{l_x} \Rightarrow {}_nd_x = l_x \cdot {}_nq_x.$$

(v) ${}_nL_x \rightarrow$ the number of members (Average) of the life table stationary pop^n in the age group $(x, x+n)$

$${}_nL_x = \int_0^n l_{x+t}\,dt = \frac{n}{2}[l_x + l_{x+n}] \quad \{\text{by Trapezoidal Rule}\}$$

(vi) $T_x \rightarrow \int_0^\infty l_{x+t}\,dt$, is the number of persons lived after age x, or the number of members of the life table stationary pop^n of age x or above.

(vii) $e_x^{\circ} = \frac{T_x}{l_x}$, complete expectation of life at age x.

(i) Reed-Merrell Method ⇒

Notations :-

(i) ${}_nq_x^z$ → is the prob' that a person who is in the age-group x to $(x+n)$ will die in the calender year Z.

$${}_nq_x^z = \frac{{}_nd_x^z}{l_x}$$

${}_nd_x^z$ → is the number of deaths in the age-group $(x, x+n)$ in the calender year Z.

$${}_nd_x^z = l_x - l_{x+n}$$

${}_nm_x^z$ → is the central Rate of mortality in the calendar year Z, n being the length of the age-group $(x, x+n)$. ${}_nm_x^z = {}_nd_x^z / {}_nP_x^z$.

${}_nP_x^z$ → is the average number of persons in the age-group x to $x+n$, in the calender year Z.

Method ⇒ this method due to L.J. Reed and M. Merrell is based on the following fundamental result which we state in the form of a lemma.

$${}_nq_x^z = \frac{2n\,({}_nm_x^z)}{2+n\,({}_nm_x^z)} \quad \text{and} \quad {}_nm_x^z = \frac{{}_nd_x^z}{{}_nP_x^z}$$

proof ⇒ let the life table popⁿ of age x in calender year z be $l_x = E_x^z$.

Assuming that deaths are uniformly distributed in the Interval $(x, x+n)$ or equivalently assuming the linearity of l_{x+t} $\forall\ t \in [0, n]$.

we get $\quad {}_nP_x^z = \int_0^n l_{x+t}\,dt \cong \frac{n}{2}(l_x + l_{x+n})$ {by Trapezodal Rule.

$$= \frac{n}{2}[l_x + l_x - {}_nd_x^z]$$

$$= n \cdot E_x^z - \frac{n}{2}({}_nd_x^z) \qquad \{ l_x = E_x^z$$

$$\Rightarrow E_x^z = \frac{1}{n} \cdot {}_nP_x^z + \frac{1}{2}\,{}_nd_x^z \quad \text{---} *$$

Now by deffinition

$${}_nq_x^z = \frac{{}_nd_x^z}{E_x^z} = \frac{{}_nd_x^z}{\frac{1}{n}\cdot {}_nP_x^z + \frac{1}{2}\,{}_nd_x^z} = \frac{{}_nm_x^z}{\frac{1}{n} + \frac{1}{2}({}_nm_x^z)}$$

$$\Rightarrow {}_nq_x^z = \frac{2n({}_nm_x^z)}{2 + n\cdot({}_nm_x^z)}$$

Note ⇒ If $n = 1$, we get

$$q_x^z = \frac{2\,m_x^z}{2 + m_x^z}$$

which is already discuss in complete life table.

$m_x \rightarrow$ central mortality rate.

Description of the Method ⇒ for each possible value of x, the
(विवरण)

values of ${}_nP_x^z$ and ${}_nd_x^z$ are Known from the Census and Registration data respectively.

using these values we can find, ${}_nm_x^z$ as

$${}_nm_x^z = \frac{{}_nd_x^z}{{}_nP_x^z}$$

and finally we find values of ${}_nq_x^z$ by the relation between ${}_nq_x^z$ and ${}_nm_x^z$.

Thus starting with given Radix l_x, we can find other values l_{x+n}, l_{x+2n}, ... by the Relation

$$l_{x+n} = l_x \cdot {}_nP_x \; ; \; l_{x+2n} = l_x \cdot {}_nP_{x+n} \; ......$$

(ii) Greville's Method ⇒

The Greville's method may be regarded as a refinement over the Reed-Merrell Method.

In this method we assume that ${}_n m_x$ follows Gompertz (exponential) Law.

So for the estimation of the ${}_n q_x$ from the observed age-specific death rate ${}_n m_x$, Greville used the Relation.

$$ {}_n q_x = \frac{2n({}_n m_x)}{2 + {}_n m_x\left[n + \frac{n^2}{6}({}_n m_x - \log_e c)\right]} \quad \text{—— (a)} $$

where c is estimated from the assumption that ${}_n m_x$ follows Gompertz Law i.e

$$ {}_n m_x = B \cdot c^x \quad ; \quad B \text{ is constant} \quad \text{—— (b)} $$

First we find the value l_x which will serve as a radix for the construction of the abridged life table.

then we find ${}_n d_x$ as ${}_n d_x = l_x \cdot {}_n q_x$ —— (i)

and we find l_{x+n} as $l_{x+n} = l_x - {}_n d_x$ —— (2)

where ${}_n d_x$ is the total number of deaths in the life table stationary pop^n in the age sector (x to x+n)

Now starting with the Radix l_x and Computing the ${}_nq_x$ Values from the relation (a). and we can obtain the values of $l_{x+n}, l_{x+2n}, \ldots$ by using eq^n (i) and (ii).

Now we calculate ${}_nL_x$ for the abridged life table. If we assume that Central death rate ${}_nm_x$ in the observed pop^n is same as in the life table stationary pop^n then by defination

$$ {}_nm_x = \frac{{}_nd_x}{{}_nL_x} \Rightarrow {}_nL_x = \frac{{}_nd_x}{{}_nm_x}. $$

where ${}_nm_x$ are given values and ${}_nd_x$ are computed from (i).

If ${}_nd_x$ is not well defined (not computed) then by definition.

$$ {}_nm_x = \frac{{}_nd_x}{{}_nL_x} = \frac{l_x - l_{x+n}}{T_x - T_{x+n}} $$

$$ = -\frac{d}{dx}\left[\log_e (T_x - T_{x+n})\right] \qquad \left\{ \frac{d}{dx} T_x = -l_x. \right. $$

$$ = -\frac{d}{dx} \log_e ({}_nL_x) $$

Integrate w.r.to x, we get.

$$ \log_e ({}_nL_x) = -\int {}_nm_x \, dx + \log K $$

$$\Rightarrow \quad {}_nL_x = K \cdot e^{-\int {}_nm_x \, dx}$$

where K is Constant.

If the assumption about ${}_nm_x$ i.e (${}_nm_x$ is same) is not Valid, then another approximation to ${}_nL_x$, based on numerical quadrature is given by formula.

$${}_nL_x = \int_0^n l_{x+t} \, dt \cong \frac{n}{2}\left(l_x + l_{x+n}\right) + \frac{n}{24}\left[{}_nd_{x+n} - {}_nd_{x-n}\right]$$

and it provides more accurate results as Compared with ${}_nL_x = {}_nd_x / {}_nm_x$.

Now if $w+n$ is the Terminal age i.e $l_{w+n} = 0$,

then $${}_nL_w = \frac{{}_nd_w}{{}_nm_w} = \frac{l_w \cdot {}_nq_w}{{}_nm_w} = \frac{l_w}{{}_nm_w} \quad \left\{ {}_nq_w = 1 \right.$$

The next column of the Table is T_x, as

$$T_x = {}_nL_x + {}_nL_{x+n} + \cdots + {}_nL_w = \sum_{i=0}^{w-x} {}_nL_{x+i}$$

Finally the Last column giving the complete Expectation of life is obtained from the Relation

$$e_x^o = \frac{T_x}{l_x}.$$

King's method ⇒ This method due to G. King is intended if the life table functions (l_x, q_x, L_x, etc) are to be obtained for the values of $x = x_0, x_0+n, x_0+2n, \ldots$ at same distance n apart say $n = 5$ years or 10 years.

In the usual notations let ${}_nP_x$ be the observed pop^n and ${}_nD_x$ be the number of deaths in age group $(x, x+n)$. Then we can write.

$$ {}_nP_x = P_{x-[(n-1)/2]} + P_{x+1-[(n-1)/2]} + \cdots + P_{x+[(n-1)/2]} $$

$$ {}_nD_x = D_{x-[(n-1)/2]} + D_{x+1-[(n-1)/2]} + \cdots + D_{x+[(n-1)/2]}. $$

where P_x and D_x are respectively the pop^n and the number of deaths for the Age-group x to $x+1$. and $[(n-1)/2]$ is the greatest Integer function value.

for example, for $n = 5$ and $x = 10$, we have.

$$ {}_5P_{10} = P_8 + P_9 + P_{10} + P_{11} + P_{12} \quad i.e $$

${}_5P_{10}$ is the total pop^n in the age group 8 to 12.

our main purpose is to obtain an estimate of the pop^n P_x° and the deaths D_x° for the central Age in the age group $(x, x+n)$, from the given values of ${}_nP_x$ and ${}_nD_x$.

Under the assumption that P_x° and D_x° can be approximated by a second degree Parabola, King obtained their estimates from the formulae.

$$\left.\begin{aligned} P_x^\circ &= \frac{1}{n}({}_nP_x) - \frac{1}{24}\left(\frac{1}{n}\right)\left(1-\frac{1}{n^2}\right)\Delta^2({}_nP_x) \\ D_x^\circ &= \frac{1}{n}({}_nD_x) - \frac{1}{24}\left(\frac{1}{n}\right)\left(1-\frac{1}{n^2}\right)\Delta^2({}_nD_x) \end{aligned}\right\} \quad — *$$

Using the estimates of P_x° and D_x° obtained from $*$, the central rate of mortality at age x is given by

$$m_x = \frac{D_x^\circ}{P_x^\circ} \quad ; \quad x = x_0, x_0+n, x_0+2n, \ldots$$

and q_x is obtained from relation

$$q_x = \frac{2m_x}{2+m_x} \quad ; \quad x = x_0, x_0+n, x_0+2n, \ldots$$

assuming that deaths are uniformly distributed over the given Interval

Now we obtain p_x, the prob^t^ of survival at age x by the relation

$$p_x = 1 - q_x \; ; \quad x = x_0, \; x_0+n, \; x_0+2n, \; \cdots$$

For the Remaining columns of the life table, we obtain l_x for the ages $x = x_0, x_0+n, x_0+2n, \cdots$ by the relation

$$l_{x+n} = l_x \cdot ({}_np_x) \; ;$$

$$l_{x+2n} = l_{x+n} \cdot ({}_np_{x+n}).$$

$$\vdots$$

where ${}_np_x$ is the prob^t^ that a person aged x survives next n years.

we Know that

$${}_np_x = p_x \cdot p_{x+1} \cdots p_{x+n-1} = \prod_{i=0}^{n-1} p_{x+i}$$

$$\Rightarrow \log({}_np_x) = \sum_{i=0}^{n-1} \log p_{x+i} \quad \text{---(i)}$$

King obtained the values of ${}_np_x$ from the available values of p_x by using Everett's Central Difference formula. as

$$U_{x+h} = \left[y\, U_{x+n} + \frac{y(y^2-1)}{3!} \Delta^2 U_x + \cdots \right] + \left[t\, U_x + \frac{t(t^2-1)}{3!} \Delta^2 U_{x-n} + \cdots \right] \quad \text{---(ii)}$$

where $0 \le h \le n$; $y = h/n$ and $t = 1-y$.

Taking $U_x = \log p_x$ and $h = 1, 2, 3, \cdots (n-1)$.

we get Correct up to second order differences

$$\log p_{x+1} = \frac{1}{n}\log p_{x+n} + \left(1-\frac{1}{n}\right)\log p_x + \frac{1}{3!}\cdot\frac{1}{n}\left(\frac{1}{n^2}-1\right)\Delta^2 \log p_x$$

$$+\frac{1}{3!}\left(1-\frac{1}{n}\right)\left[\left(1-\frac{1}{n}\right)^2-1\right]\Delta^2 \log p_{x-n}$$

taking $h=1$, $y=\frac{1}{n}$. $t=1-y=1-\frac{1}{n}$ in E^n (ii)

⋮

$$\log p_{x+n-1} = \frac{n-1}{n}\log p_{x+n} + \left(1-\frac{n-1}{n}\right)\log p_x + \frac{1}{3!}\left(\frac{n-1}{n}\right)\left[\left(\frac{n-1}{n}\right)^2-1\right]\Delta^2 \log p_x$$

$$+\frac{1}{3!}\left(1-\frac{n-1}{n}\right)\left[\left(1-\frac{n-1}{n}\right)^2-1\right]\Delta^2 \log p_{x-n}$$

taking $h=n-1$, $y=\frac{h}{n}=\frac{n-1}{n}$; $t=1-y=1-\frac{(n-1)}{n}$ in (ii)

$$\Rightarrow \log({}_n p_x) = \sum_{r=0}^{n-1}\log p_{x+r} = \frac{n+1}{2}\log p_x + \frac{n-1}{2}\log p_{x-n}$$

$$-\frac{n^2-1}{24n}\left(\Delta^2 \log p_x - \Delta^2 \log p_{x-n}\right) \quad (3)$$

{ for simplifying

King's obtained the values of ${}_np_x$ from E^n (3) and then using $l_{x+n} = l_x\cdot({}_np_x)$ obtained the values of l_x for $x = x_0, x_0+n, x_0+2n, \ldots\ldots$

Now we calculate $T^*_{x:n}$ as

$T^*_{x:n}$ = number of years lived by the Radix l_x during the age Interval $[x, x+n]$

$$\Rightarrow T^*_{x:n} = L_x + L_{x+1} + \cdots + L_{x+n-1}.$$

$$= \frac{1}{2}(l_x + l_{x+1}) + \frac{1}{2}(l_{x+1}+l_{x+2}) + \cdots + \frac{1}{2}(l_{x+n-1}+l_{x+n})$$

$$= (l_x + l_{x+1} + l_{x+2} + \cdots l_{x+n-1}) - \frac{1}{2}(l_x - l_{x+n})$$

$$= \sum_{i=0}^{n-1} l_{x+i} - \frac{1}{2} l_x \left(1 - \frac{l_{x+n}}{l_x}\right) = \sum_{i=0}^{n-1} l_{x+i} - \frac{1}{2} l_x (1 - n p_x)$$

on using Eq^n (3) with $\log p_x$ replaced by l_x

we get

$$T^*_{x:n} = \frac{n+1}{2} l_x + \frac{n-1}{2} l_{x+n} - \frac{n^2-1}{24n}\left(\Delta^2 l_x + \Delta^2 l_{x-n}\right) - \frac{1}{2} l_x (1 - n p_x)$$

$$\Rightarrow T^*_{x:n} = N^*_{x:n} - \frac{1}{2} l_x \cdot n q_x$$

where $N^*_{x:n} = \sum_{i=0}^{n-1} l_{x+i}$

$$= \frac{n+1}{2} l_x + \frac{n-1}{2} l_{x+n} - \frac{n^2-1}{24}\left(\Delta^2 l_x + \Delta^2 l_{x-n}\right)$$

is the total number of complete years lived by l_x persons having aged x to $(x+n)$.

Thus $T^*_{x:n}$ can be obtained for $x = x_0, x_0+n, x_0+2n, \dots$

Finally, $e^0_x = \frac{T^*_{x:n}}{l_x} = \frac{N^*_{x:n}}{l_x} - \frac{1}{2} \cdot n q_x$.

Fertility ⇒ Here fertility means Actual production of children or occurrence of births, specially live births.

Fertility must be different from fecundity which refers to the capacity to bear children. In fact, fecundity provides an upper bound for fertility.

As a measure of the Rate of growth of popn Various fertility Rates are computed.

(i) Crude Birth Rate (C.B.R) ⇒

this is the simplest of all the measures of fertility and consists in relating the number of live births to the total popn.

$$C.B.R = \frac{B^t}{P^t} \times K$$

$B^t \rightarrow$ Total no of live births in the given Region during a given period of Time.

$P^t \rightarrow$ Total popn of the given Region during the period t.

$K = 1000$ (usually)

Merits ⇒ It is simple, easy to calculate. it is based only on the number of births (B^t) and the total size of the popn (P^t).

Demerits ⇒

(i) It completely Ignores the sex-distribution of the popn.

(ii) C.B.R is not a probt ratio (probt), since the whole popn P^t can not be regarded as exposed to the Risk of producing children. In fact, only the females and only those between the child bearing age-group (15-49) are exposed to risk and as such whole of the male popn and the female popn outside the child-bearing age should be Excluded from P^t, but Here not.

(iii) C.B.R assumes that women in all the ages have the same fertility, an assumption which is not true since younger woman have, in general Higher fertility than elderly women.

General Fertility Rate (G.F.R) ⇒

This consists in relating the total no of live births to the number of females in the reproductive or child bearing ages.

$$G.F.R = \frac{B^t}{\sum_{x=\lambda_1}^{\lambda_2} f_{P_x}} \times K$$

$B^t \rightarrow$ number of live births occurring among the pop^n of a given geographic area during a given period t.

$\sum_{\lambda_1}^{\lambda_2} f_{P_x} \rightarrow$ Total female pop^n in the reproductive age, in the given geographical Region during the same time t.

$\lambda_1, \lambda_2 \rightarrow$ Lower and upper limits of the female child bearing age,

$K = 1000.$

NOTE ⇒ In general $\lambda_1 = 15$ and $\lambda_2 = 49.$

Merits ⇒

(i) G.F.R is a prob' rate since the denominator consists of the entire female popⁿ which is exposed to the Risk of producing children.

(ii) G.F.R reflects the extent to which the female popⁿ in the reproductive ages increases the existing popⁿ through live births.

Demerits ⇒ G.F.R gives a Heterogeneous figure since it overlooks the age composition of the female popⁿ in the child-bearing age. Hence it suffers from the drawback of non-comparability in respect of Time and Country i.e. The two populations with same G.F.R. may exhibit entirely different fertility ~~Rate~~ status.

Specific Fertility Rate (S.F.R) ⇒

Age-Specific Fertility Rate ⇒

The fertility rate for different Age-groups of reproductive age separately is called the Age-specific fertility rate.

The Age-specific fertility rate for the age-group x to $x+n$, denoted by ${}_n i_x$ is defined as

$$ {}_n i_x = \frac{{}_n B_x}{{}_n^f P_x} \times K $$

${}_n B_x$ → Number of births to the females in the age group $[x, x+n)$ in the given geographic Region during a period t.

${}_n^f P_x$ → Average female pop^n of ages $[x, x+n)$ in the given area during the period t.

$K = 1000$.

NOTE: In particularly, If $n=1$, we get Annual age-specific fertility rate

$$ i_x = \frac{B_x}{{}^f P_x} \times K. $$

NOTE ⇒ Age-specific fertility rate is a proper rate; it removes the drawback of G.F.R. by taking into account the Age-Composition of the women in the child-bearing age group and is thus suitable for comparative studies.

However, the use of age-S.F.R for comparing the fertility situations of two regions or of the same Region for two different periods is not an easy job.

Total Fertility Rate (T.F.R) ⇒ As already pointed out that, age-specific fertility Rate is not of much practical utility for comparative purposes.

In order to arrive at some more practical measure of the pop^n growth, the age specific fertility rates for different age-groups have to be combined together to give a single quantity.

This leads to total fertility rate T.F.R which is obtained on adding the annual age-specific fertility rates

$$\Rightarrow \quad T.F.R = \sum_{\lambda_1}^{\lambda_2} i_x = \sum_{\lambda_1}^{\lambda_2} \frac{B_x}{{}^{f}P_x} \times K \quad \text{———} *$$

Usually $\lambda_1 = 15$ and $\lambda_2 = 49$. Thus in order to compute T.F.R from $*$, we shall have to calculate 34 age-specific fertility rates.

The arithmetic (calculations) may be reduced to a great extent by working with age-groups say, x to $x+n$, where in general n, the width of Interval may vary from one group to the other group.

In such a case, the T.F.R is approximately given by

$$T.F.R = \sum_x n \cdot ({}_n i_x)$$

Measurement of pop^n Growth ⇒ Now we measure the pop^n growth rate

Since Fertility rates are Inadequate (अपर्याप्त) to give us any Idea about the rate of pop^n growth because, they Ignore the Sex (boy or girls) of the newly born children and their mortality. and this is a Serious problem because pop^n increases through female births

Now we study some measures of the growth of pop^n.

Crude Rate of Natural Increase and Pearle's Vital Index ⇒

The Simplest measure of the pop^n growth Known as crude rate of natural increase is defined as the difference between the Crude birth Rate (per thousand) and Crude death rate (per thousand)

⇒ Crude Rate of Natural Increase = C.B.R − C.D.R

this formula gives the Net increase (decrease) in pop^n through births and deaths taken together.

Another Indicator of pop^n growth is Pearle's Vital Index, defined as

$$\text{Pearle's Vital Index} = \frac{C.B.R}{C.D.R} \times 100$$

NOTE ⇒ Pearle's Vital Index merely (केवल) gives a measure whether births exceed deaths or not. it does not tell us anything whether pop^n has a tendency to increase or decrease.

NOTE ⇒ Both these measures suffer from the drawbacks of C.B.R and C.D.R and as such are not suitable for Comparative studies.

Gross Reproduction Rate (G.R.R) ⇒
(female).

In order to have a better Idea about the rate of popⁿ growth we must take into account the sex of the new born children since it is ultimately the female births who are the potential future mothers and result in an increase in the popⁿ.

The gross Reproduction Rate (G.R.R) is a step in this direction and is defined as the sum of age-specific fertility rates calculated from female births for each year of Reproductive period.

If ${}^{f}B_x$ is the number of female births to the women of age x during the given period in the given region.

${}^{f}P_x$ is Total female popⁿ of age x. then

$$G.R.R = \sum_{d_1}^{d_2} \frac{{}^{f}B_x}{{}^{f}P_x} \times k = \sum_{d_1}^{d_2} {}^{f}i_x$$

where ${}^{f}i_x = \frac{{}^{f}B_x}{{}^{f}P_x} \times k$ is termed as the female age-specific fertility rate.

this is also called female G.R.R.

Gross reproduction Rate is thus a modified form of Total fertility Rate.

Suppose now that instead of annual data we are given the figures for different age groups of reproductive period. Let ${}_n^fB_x$ be the number of female babies born to the women, in the age-group x to $x+n$. then

$$G.R.R = \sum_{x_1}^{x_2} n\left(\frac{{}^fB_x}{{}_n^fP_x}\right)\times k = \sum_{x_1}^{x_2} n\left({}_n^fi_x\right)$$

where ${}_n^fi_x$ is the age-specific fertility rate for the age-group x to $x+n$. based on female births.

Limitation ⇒ G.R.R is computed on the Hypothesis that none of the newly born female babies is subject to the risk of mortality till the end of the reproductive period of life. this is a Very Serious limitation of G.R.R.

Net Reproduction Rate (N.R.R) ⇒ As pointed out that the principle limitation of G.R.R is that it completely Ignores the current mortality and takes into account only the current fertility.

Net Reproduction Rate (N.R.R) is nothing but gross reproduction Rate (G.R.R) adjusted for the effect of mortality.

N.R.R measures the extent to which a generation of girls babies survive to reproduce themselves as they pass through the child-bearing age group.

Let us now take into consideration the factor of mortality of mothers also in measuring the growth of pop^n.

Notations ⇒

${}_n^fL_x$ → mean size of the Redix ${}^f l_0$ females in the age-Interval x to $x+n$.

${}_n^fB_x$ → number of female births to the women in the age group x to $x+n$. at any point t.

then $\frac{{}_n^fL_x}{{}^f l_0} \times {}_n^fB_x$ gives the average number of female children that would be born to the chort (Radix) $^f l_0$ in the age-group x to $x+n$.

The quantity
$$ {}_n^f\pi_x = \frac{{}_n^f L_x}{{}^f l_0} $$

gives the life table probability of survival of a female to the age-Interval x to $x+n$.

and is called the Survival rate.

$\Rightarrow$ out of K newly born female babies $K \times ({}_n^f\pi_x)$ will enter into the child bearing age-Interval x to $x+n$. and

$K \times ({}_n^f\pi_{x+n})$ into the age-group $x+n$ to $x+2n$. and so on.

Hence female Net reproduction rate (N.R.R) is given by

$$ N.R.R = K \sum_x n \left[\frac{{}_n^f B_x}{{}_n^f P_x} \times {}_n^f\pi_x \right] $$

$$ = K \sum_x \left[n \, ({}_n i_x) \times {}_n^f\pi_x \right] $$

$$ = K \sum_x \left[n \times \text{female Age-S.F.R} \times \text{Survival factor} \right] $$

Graduation of Mortality Rates ⇒
(क्रम से आगे की ओर बढ़ना) (Smoothing of the f^n μ_x) ⇒

The computation of the age-S.D.R's m_x for any pop^n from the causes data or sample registration data is subject to a number of irregularities.

In order to use these rates for further mathematical work, specially in the construction of life tables, we should smooth out (continue) these irregularities.

Hence we need to obtain some explicit expressions (स्पष्ट) for m_x in terms of x.

we try to obtain an explicit expression for μ_x, the force of mortality at age x. which is related to m_x as

$$m_x \doteqdot \mu_{x+\frac{1}{2}}$$

NOTE:- A number of attempts have been made to develop a suitable formula for μ_x, from time to time but the most successful of them seems to be given by Makeham.

Makham's Graduation Formula ⇒

Makham's assumes that deaths occur to two causes (i) Accidents (ii) Diseases.

He further assumes that

(a) The effect of Accidents is constant throughout the life span.

(b) The capacity of the Human body to resist diseases decreases as age increases. i.e.

The force of mortality would vary inversely as some f^n of age x, say $g(x)$., which represents the force of resistance (प्रतिरोध) to disease.

$$\Rightarrow \quad \mu_x = A + \frac{B}{g(x)} \quad \text{---}* \quad ; A>0 \quad ; B>0 \quad ; g'(x)<0$$

Makeham further assumes that in a short span, a person loses a constant proportion $r>0$, of such force of resistance to disease as he still has.

⇒ Makeham takes, Instant rate of decrease in $g(x)$ = Constant.

$$\Rightarrow \quad -\frac{1}{g(x)} \cdot \frac{d}{dx}\{g(x)\} = r \quad \Rightarrow \frac{1}{g(x)} \cdot \frac{d}{dx}\{g(x)\} = -r$$

Integrate w.r.to x, we get

$$\log g(x) = -rx + c_1$$

$$\Rightarrow g(x) = e^{c_1 - rx} = c_2 e^{-rx} \quad ; \quad c_2 = e^{c_1}$$

put in $*$, we get

$$\mu_x = A + \frac{B}{c_2 e^{-rx}} = A + D c^x \quad \begin{cases} D = \frac{B}{c_2} \\ c = e^{r} \end{cases}$$

$\Rightarrow$ Since $\mu_x = -\frac{1}{l_x} \cdot \frac{dl_x}{dx}$

$$\Rightarrow -\frac{1}{l_x} \frac{d}{dx}(l_x) = A + Dc^x$$

Integrate w.r. to x, we get

$$\log(l_x) = -\left[Ax + \frac{Dc^x}{\log_e c} + E\right]$$

$$\Rightarrow l_x = e^{-E} \cdot e^{-Ax} \cdot e^{-Dc^x/\log_e c}$$

$$\Rightarrow l_x = K \cdot S^x \cdot p^{c^x} \quad \text{—— (i)} \quad \begin{cases} K = e^{-E} \\ S = e^{-A} \\ p = e^{-D/\log_e c} \end{cases}$$

Here K, S, p, c are called four parameters of l_x the formula (i) known as Makeham's formula, is used to graduate (smoothing) the l_x figures in a life table, from which the force of mortality μ_x can be computed.

NOTE: $0.001 < A < 0.003$; $10^{-6} < D < 10^{-3}$; $1.08 < c < 1.12$.

Gompertz Makeham Graduation formula for Mortality $\Rightarrow$ Gompertz give the idea of mortality by considering only the force of resistance to diseases exactly in the same manner as Makeham did but He completely Ignored the factor of accidents.

this leads to

$$A = 0 \quad \Rightarrow \quad s = e^{-A} = 1$$

$\Rightarrow$ Gompertz graduation formula for l_x becomes.

$$l_x = K \cdot p^{c^x} = K \cdot p^{c^x}$$

$\Rightarrow$ Gompertz mortality Law is given by

$$\mu_x = D \cdot c^x \qquad \{A = 0.$$

Fitting of Makeham's Graduation formula ⇒

In this section, we will discuss the fitting of Makeham's formula to the given set of data, to the l_x function, assuming that the data related to the l_x rather than μ_x.

Since the l_x function involves 4 parameters, K, S, C, P, we need four independent equations to determine them.

I: Method of four Selected Points ⇒

These four parameters will be so determined that the relating curve passes through the four equally spaced points.

let these chosen points be $x = 0, n, 2n, 3n$.

Now taking log of eq^n $l_x = K \cdot S^x \cdot P^{C^x}$

$$\Rightarrow \log l_x = \log K + x \log S + C^x \log P \quad -(2)$$

$$\text{So } \log l_0 = \log K + 0 + \log P \quad -(a)$$

$$\log l_n = \log K + n \log S + C^n \log P \quad -(b)$$

$$\log l_{2n} = \log K + 2n \log S + C^{2n} \log P \quad -(c)$$

$$\log l_{3n} = \log K + 3n \log S + C^{3n} \log P \quad -(d)$$

$$\Rightarrow \Delta l_x = l_{x+n} - l_x$$

$$\Rightarrow \Delta \log l_x = \log(l_{x+n}) - \log(l_x)$$

$$\Rightarrow \Delta \log l_x = [\log K + (x+n)\log s + c^{x+n}.\log p] - [\log K + x\log s + c^x \log p]$$

$$\Rightarrow \Delta \log l_x = n\log s + c^x(c^n-1)\log p = V_x \text{ (let)}. \quad \text{—(3)}$$

$$\Rightarrow \Delta^2 \log l_x = \Delta V_x = V_{x+n} - V_x.$$

$$\Rightarrow \Delta^2 \log l_x = [n\log s + c^{x+n}(c^n-1)\log p] - [n\log s + c^x(c^n-1)\log p]$$

$$\Rightarrow \Delta^2 \log l_x = c^x(c^n-1)^2.\log p \quad \text{—(4)}$$

taking $x = 0, n,$ and $2n$ in Eq^n (3) and (4), get

$$\left.\begin{aligned} \Delta \log l_0 &= n\log s + (c^n-1)\log p \\ \Delta \log l_n &= n\log s + c^n(c^n-1)\log p \\ \Delta \log l_{2n} &= n\log s + c^{2n}(c^n-1)\log p \end{aligned}\right\} \quad \text{—(a')}$$

and

$$\left.\begin{aligned} \Delta^2 \log l_0 &= (c^n-1)^2\log p \\ \Delta^2 \log l_n &= c^n(c^n-1)^2\log p. \end{aligned}\right\} \quad \text{—(b')}$$

$$\Rightarrow c^n = \frac{\Delta^2 \log l_n}{\Delta^2 \log l_0} \quad \text{—(c')}$$

the values of $\Delta^2 \log l_n$ and $\Delta^2 \log l_0$ can be obtained from the given set of 4 points (x, l_x), $x = 0, n, 2n, 3n$ on completing the difference table. as

x	l_x	$\log l_x$	$\Delta \log_x$	$\Delta^2 \log_x$
0	l_0	$\log l_0 = a$ (let)	$b-a = P$ (let)	$Q-P = U$ (let)
n	l_n	$\log l_n = b$	$c-b = Q$	$R-Q = V$
$2n$	l_{2n}	$\log l_{2n} = c$	$d-c = R$	
$3n$	l_{3n}	$\log l_{3n} = d$		

by Eq^n (c') we find value of c.
put value of c^n in Eq^n (b') (first Eq^n)
we get p.
put values of c^n and p in first Eq^n of (a')
we get estimated value of s.
and finally we get k from Eq^n (a).

II Method of Partial Sums ⇒

This is an Improvement over the method of 4 selected points.

In order to use the entire data, we use the method of partial sums which consists in dividing the entire series into 4 equal parts separately and find the sums for each part as given below.

Let the four equal parts of the series be

Part-I : $x = 0, 1, 2, \ldots (n-1)$

Part-II : $x = n, n+1, n+2, \ldots (2n-1)$

Part-III : $x = 2n, 2n+1, 2n+2, \ldots (3n-1)$

Part-IV : $x = 3n, 3n+1, 3n+2, \ldots (4n-1)$

$$\text{let } S_0 = \sum_{x=0}^{n-1} \log l_x \;;\quad S_1 = \sum_{x=n}^{2n-1} \log l_x \quad \text{—(1)}$$

$$S_2 = \sum_{x=2n}^{3n-1} \log l_x \;;\quad S_3 = \sum_{x=3n}^{4n-1} \log l_x .$$

$$\Rightarrow S_0 = \sum_{x=0}^{n-1} \left[\log k + x \log s + c^x \log p\right]$$

$$= n \log k + \log s \{1+2+3+\cdots+(n-1)\} + \log p \{1+c+c^2+\cdots+c^{n-1}\}$$

$$\Rightarrow S_0 = n \log k + \log s \cdot \frac{n(n-1)}{2} + \log p \cdot \frac{c^n - 1}{c-1} \quad \text{—— (2)}$$

Now

$$S_1 = \sum_{x=n}^{2n-1} \left[\log k + x \log s + c^x \log p\right]$$

$$\Rightarrow S_1 = n\log K + \log s\{n+(n+1)+\cdots+(2n-1)\} + \log p\{c^n + c^{n+1} + \cdots + c^{2n-1}\}$$

$$= n\log K + \log s \cdot \frac{n}{2}(3n-1) + \log p \cdot \frac{c^n(c^n-1)}{c-1} \quad \text{—(3)}$$

Similarly

$$S_2 = n\log K + \log s \cdot \frac{n}{2}(5n-1) + \log p \cdot \frac{c^{2n}(c^n-1)}{c-1} \quad \text{—(4)}$$

$$S_3 = n\log K + \log s \cdot \frac{n}{2}(7n-1) + \log p \cdot \frac{c^{3n}(c^n-1)}{c-1} \quad \text{—(5)}$$

taking Successive differences of these partial Sums

$$\Delta S_0 = \sum_{x=0}^{n-1} \Delta \log l_x = \sum_{x=0}^{n-1}[n\log s + c^x(c^n-1)\log p] \quad \left\{\begin{array}{l}\Delta \log l_x \\ = n\log s + c^x(c^n-1)\log p\end{array}\right.$$

$$= n^2\log s + \left[\frac{(c^n-1)^2}{c-1}\right]\log p$$

$$\Delta S_1 = \sum_{x=n}^{2n-1} \log l_x = \sum_{x=n}^{2n-1}[n\log s + c^x(c^n-1)\log p]$$

$$= n^2\log s + \log p \cdot c^n\left[\frac{(c^n-1)^2}{c-1}\right]$$

$$\Delta S_2 = n^2\log s + \log p \cdot c^{2n}\left[\frac{(c^n-1)^2}{c-1}\right]$$

Now $\Delta^2 S_0 = \sum_{x=0}^{n-1} \Delta^2 \log l_x = \sum_{x=0}^{n-1}[c^x(c^n-1)^2\log p]$

$$\Rightarrow \Delta^2 S_0 = \log p \cdot \frac{(c^n-1)^3}{c-1}$$

Similarly $\Delta^2 S_1 = \log p \cdot \frac{c^n(c^n-1)^3}{c-1}$.

$$\Rightarrow c^n = \frac{\Delta^2 S_1}{\Delta^2 S_0}$$

put the Value of c in the preceding Eq^n
we get p, s, k.

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III **Makeham's Second Law of Mortality** $\Rightarrow$

Makeham's suggested a second modification as

$$\mu_x = A + Gx + Dc^x.$$

which consists of the sum of two parts, one linear curve and other exponential curve.

$$\Rightarrow \quad -\frac{1}{l_x}\frac{d}{dx}(l_x) = A + Gx + Dc^x$$

Integrating w.r.t. x.

$$\log_e l_x = -\left[Ax + G\cdot\frac{x^2}{2} + D\cdot\frac{c^x}{\log_e c}\right]$$

$$= -Ax - \frac{1}{2}G\cdot x^2 - \frac{Dc^x}{\log_e c} + \log_e k$$

$$\Rightarrow \quad l_x = k\cdot(e^{-A})^x\cdot\left(e^{-G/2}\right)^{x^2}\cdot\left(e^{-D/\log_e c}\right)^{c^x}$$

$$\Rightarrow \quad l_x = k\cdot s^x\cdot w^{x^2}\cdot p^{c^x}$$

Migration ⇒ movement of people across a specified boundary for purpose of establishing a new residence.

Migration is classified as either Internal migration or International migration

In and out Migration ⇒ The process of entering one administrative subdivision of a country from another subdivision is known as In-migration and Any migration from specified area to out-side is out-migration.

Gross and Net Migration ⇒ The total movement in a specified area in given time period is gross migration.

$$\Rightarrow \text{Gross } m = \text{In-}m + \text{out-}m.$$

The Net effect of In-m and out-m on areas pop^n in given Time period.

$$\Rightarrow \text{Net-}m = \text{In-}m - \text{out-}m.$$

Internal Migration ⇒

(i) Rural to Rural migration ⇒ it is of great Volume and Significance, especially among females who move primarily due to marriages or other familiar Reasons.

(ii) Rural to Urban migration ⇒ It is the most important internal migration as it contributes to transfer of Labour force from traditional agricultural sector to Urbanized industrial sector in seek of opportunities and employment. it is linked with process of Urbanization.

(iii) Urban to Rural ⇒ Past Retirement people.

(iv) Urban to Urban ⇒ work, opportunities.

Determinants of Internal Migration ⇒

The factors determining migration may be classified into 3 broad categories.

(i) Economic ⇒ when individual migrates to a place where one can aspire to have a career and better job opportunities.

(ii) Social ⇒ when individual migrates to have a higher standard of living.

(iii) Demographic ⇒ Migration due to some demographic change.

Net Migration Rate ⇒ It is the difference between no of immigrations and no of Emigrations throughout the year.

it is calculated over one year period by using mid year population

$$\Rightarrow N = \frac{I - E}{m \times 1000} \quad ; \quad m \rightarrow \text{Mid year pop}^n$$

If $I > E \rightarrow$ will have +ve Net migration Rate

$I < E \rightarrow$ " " −ve " "

$I = E \rightarrow$ " " balanced " "

Measurements of Internal Migration ⇒

A-: Direct method.

(i) place of Birth method

(ii) Duration of Residence method.

(iii) place of Last Residence

(iv) place of residence at a fixed period or (prior date).

B-: Indirect method

(i) Vital statistics method

(ii) Survival Ratio method

(iii) Reverse method.

(iv) Average method.

(v) Migration Rate method.

A-I ⇒ place of Birth Method ⇒ place of birth gives Information about migrant and Non-migrants.

Migrant -: A person enumerated (सूचीबद्ध) in a place which is not their place of birth.

Non-Migrant -: A person enumerated in a place where they are born.

In census, a question is asked directly about the place of birth and according the person is classified.

A-II ⇒ Duration of Residence Method ⇒

How Long have you been living in this place. A direct question is asked in census. on the basis of this question people are distinguished as those born outside the area of enumeration. those born in the area of enumeration. This method also takes in Account the no of return migrants.

A-III ⇒ place of Last Residence Method ⇒

People Can be easily classified as migrants whenever their place of last residence and current residence differs.

A-IV ⇒ place of Residence at a fixed prior date:

Under this method a person whose place of residence at a fixed prior date is different from place of enumeration is considered as migrant.

Indirect methods ⇒

These methods are used for measuring the volume of migration i.e no of migrants in a pop^n during a given period.

B-I → Vital statistics method ⇒ In a country that has reliable data on pop^n size, births, deaths, net migration can be estimated using the following Equation.

$$I - E = (P_z - P_1) - (B - D)$$

P_z → pop^n at z^{th} year
P_1 → " " 1^{st} "
B → No of Births in a year (Average birth)
D → " " Deaths " " "
I → " " Immigrants.
E → " " Emigrants.

This method is also known as Residual method.

B-II → Survival Ratio method ⇒

(a) Forward method ⇒ Even in absence of death statistics, net migration can be estimated if information on probability of survival is known.

If P_x^z persons belonging to age group $(x, x+1)$ at calender year z. and ${}_nS_x^z$ is the probability of surviving next n-years.

Then Expected no of person aged x+n at n years later would be

$$\hat{P}_{x+n}^{z+n} = {}_nS_x \cdot P_x^z .$$

If there is a migration the Eq^n will not balance and difference can be attributed due to net Migration.

$$\Rightarrow {}_nN.m_x^f = P_{x+n}^{z+n} - {}_nS_x \cdot P_x^z \; ; \; {}_nN.m_x^{(f)} \rightarrow \text{forward method}$$

This eq^n can be applied for male and female pop^n separately also.

This method of obtaining an estimate of net migration is called the forward survival Ratio method.

The survival probability ${}_nS_x$ would be either the life table survival ratio or one obtained from census data.

Limitations ⇒

i) Net migration in the age group (0-n) can not be estimated without estimating births.

Estimating births is itself a problem as births to migrants can not be estimated separately. Also the estimates would be accurate if volume of net migration is uniformly spread over n years.

(ii) If In-migration > out-migration and is concentrated at the early part of the interval then the estimate obtained by forward method would underestimate as part of immigrants would have either died or migrated back.

B-III ⇒ Reverse Method ⇒ To overcome the bias of greater immigration at early part of the Interval, it may be advisable to start estimation from the later time point and estimate size of pop^n at precious time.

This approach is known as Reverse Survival Ratio method.

We have estimate of Net Migration as

$$ {}_nNM_x^{r} = \frac{P_{x+n}^{z+n}}{{}_nS_x} - P_x^{z}. $$

Thus functional Relation between forward and Reverse estimates is

$$ {}_nNM_x^{r} = \frac{{}_nNM_x^{f}}{{}_nS_x} $$

The difference between Two estimates could be narrow if ${}_nS_x \to 1$

B-IV ⇒ Average Method ⇒ The problem here arises is to choose among the Two methods. one way of over comming this difficulty is to take the Average of Two estimates.

$$ {}_nNM_x^{a} = \frac{{}_nNM_x^{f} + {}_nNM_x^{r}}{2} $$

B-V ⇒ Migration Ratio Method ⇒

The Migrate Ratio means the % of pop^n migrating during the particular Time period. it can be found out by using

$$ m = \frac{M_t}{P_t} \times K. $$

$m \rightarrow$ Rate of Migration

$M_t \rightarrow$ no of Migrants in popⁿ P_t at time t
or during time Interval t.

$k = 100$.

Both the Selection of appropriate rate bases i.e M_t and P_t and the interpretation of rates depend upon the nature of available data, eg - How a migrant is defined.

Migration Models ⇒

1. Reason for movement.
2. Pattern of movement.

1-a ⇒ Lee's Migration Model ⇒ Lee's Migration model describes the push and pull factors of migration which are basically reasons for emigration and Immigration.

A push factor is something that is unfavorable about the area and is a reason for leaving that area.

A pull factor is a factor that attracts someone into an area.

The factors could be Economic Cultural and environmental.

1-b ⇒ Gravity Model ⇒ It is derived from Newton's Law of Gravity which states "Any Two bodies attract one another with a force that is proportional to the product of their masses and Inversely proportional to the Square of the distance between them.

when used in migration the words body and masses are replaced by Locations and pop^n of that area.

In Simple words big things attract each other more than do small objects and things close to each other have stronger mutual attractions than do objects at greater distance.

most Migrants move relatively short distances.

1-c ⇒ Harris's Todaro Model ⇒ The Harris-Todaro model of the rural-Urban migration process is revisited under an agent-based approach. The migration of the workers is interpreted as a process of social learning by imitation, formalized by a computational model.

2-a ⇒ Step Migration ⇒ Rural → Towns → Cities → Metro-politan area.

2-b ⇒ Circular Migration ⇒

Rural → Cities (earn money) → Rural.

International migration ⇒ The movement of a person or group from their country of birth or residence to another country for work, as a tourist, Higher studies or for bussiness etc.

Classification ⇒
(i) Permanent Migration
(ii) Labour migration
(3) Brain drain (प्रतिभा पलायन)
(4) Refugee Migration
(5) Illegal migration.

Determinants ⇒
(i) Social and Cultural
(ii) Political
(iii) Economical
(iv) Demographical

International migration is of major concern for pop^n planners, Social scientist, policy makers.
The Rise of global mobility, increasing complexity of migratory patterns and its impact on migrants and families have all contributed to International migration becoming a priority for International Community.

International organization for migration (IOM) is the international body that provides services and advice concerning migration to government and people.

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population projection and Estimates ⇒

Estimates of pop^n are of three Types.

(i) Inter - censal estimate of pop^n corresponding to time period between Two past censuses.

(2) Post - censal estimate corresponding to time point in past but subsequent to latest census.

(3) A projection corresponding to time period in future.

Inter - Censal and Post - censal Estimates ⇒

(i) Mathematical Approach ⇒

let $t=0$ and $t=1$ be the time at which last two census were held.

If we assume linear growth for pop^n then we have

$$P_t = a + bt$$

$$\Rightarrow \quad P_0 = a \quad \text{and} \quad P_1 = a + b$$

$$\Rightarrow \quad P_t = P_0 + (P_1 - P_0)t \quad \text{--- (i)}$$

If we assume exponential Growth.

$$P_t = ab^t$$

$$\Rightarrow \quad P_0 = a \quad \text{and} \quad P_1 = ab$$

$$\Rightarrow \quad P_t = P_0 \left(\frac{P_1}{P_0}\right)^t \quad \text{--- (2)}$$

Equations (1) and (2) will give Inter-censal estimates if $0 < t < 1$ and Post censal if $t > 1$.

(2) Component Approach ⇒

If $B^{(0-t)}$, $D^{(0-t)}$, $E^{(0-t)}$, $I^{(0-t)}$ be the no of births, deaths, Total Emigration and immigration in the Time period $(0-t)$ then the Inter-censal estimates are obtained as

$$P_t = B^{(0-t)} + P_0 - D^{(0-t)} + I^{(0-t)} - E^{(0-t)} \quad \text{--- (1)}$$

In the absence of error it will give the true value of P_t, the difference between the census value of P_1 and value of P_1 obtained from eq^n (1) is known as error of closure.

one can improve it by adding the fraction of error of closure to (1).

For Post-censal estimates we have the Eq^n.

$$P_t = P_1 + B^{(1-t)} - D^{(1-t)} + I^{(1-t)} - E^{(1-t)} \quad \text{--- (2)}$$

where $B^{(1-t)}$, $D^{(1-t)}$, $I^{(1-t)}$, $E^{(1-t)}$ are the number of births, deaths, Total Immigration and Emigration in Time period $(1-t)$.

Projection method ⇒

(1) Thomas Prakash method of popⁿ Projection ⇒

It consists in estimating 3 components responsible for changes in popⁿ separately and then combining them. it consists in estimating.

(1) Survivors of base year popⁿ to year of projection (mortality factor).

(2) No of births from base year to year of projection and their Survival (fertility factor).

(3) Migration component.

(1) Let us Suppose that projections are made on t^{th} year. Further let

${}_nP_x^z \rightarrow$ popⁿ in age Interval x to $x+n$ in calender year z.

${}_nP_x^{z+t} \rightarrow$ " " $z+t$

also we know that

$$\frac{{}_nP_x^{z+t}}{{}_nP_x^z} = \frac{{}_nL_x^{z+t}}{{}_nL_x^z}$$

$$\Rightarrow {}_nP_x^{z+t} = \frac{{}_nL_x^{z+t}}{{}_nL_x^z} \cdot {}_nP_x^z$$

(2) Suppose $t = nk$ be multiple of n. and the Interval $(0, t)$ Can be divided into K Sub-Intervals

$0 \quad n \quad 2n \quad t-2n \quad t-n \quad t$

Consider particular year $t-n+i$ in the Interval $t-n$ to t.

let $B^{t-n+i} \rightarrow$ no of births in $t-n+i^{th}$ year and B^{t-n+i} are exposed to risk of mortality upto time t i.e an average $t-(t-n+i)+\frac{1}{2}$ year i.e $(n-i)+\frac{1}{2}$ year

$\Rightarrow$ Expected no of births who will Survive at time

$$T=t \quad = B^{t-n+i} \cdot \underset{\substack{\downarrow \\ \text{prob}^t \text{ of Surviving} \\ n-i+\frac{1}{2} \text{ year.}}}{P_{n-i+\frac{1}{2}}} = B^{t-n+i} \cdot \frac{l_{n-i+\frac{1}{2}}}{l_0} = B^{t-n+i} \cdot \frac{L_{n-i}}{l_0}$$

$\Rightarrow$ Expected no of Survivors of Birth in $(t-n, t)$

$$= \sum_{i=0}^{n} B^{t-n+i} \cdot \frac{L_{n-i}}{l_0}$$

and Expected no of Survivors of birth in $t-2n$ to $t-n$ is

$$\sum_{i=0}^{n} B^{t-2n+i} \cdot \frac{L_{2n-i}}{l_0}$$

Expected no of Survivors of birth in $t-kn$ to $t-(k-1)\cdot n$.

is $$\sum_{i=0}^{n} B^{t-kn+i} \cdot \frac{L_{kn-i}}{l_0}$$

Thus expected no of Survivarse of birth during $(0, t)$ is

$$\sum_{i=0}^{n}\left[B^{t-n+i}\cdot\frac{L_{n-i}}{l_0}+B^{t-2n+i}\cdot\frac{L_{2n-i}}{l_0}+\cdots+B^{t-nk+i}\cdot\frac{L_{nk-i}}{l_0}\right]$$

(3) Regarding estimation of migration components it is assumed that current migration trend will continue in future.

To simplify the computation, net migration during the period of time t years. may be assumed to be concentrated on last day of period.

In this way no account of births and deaths is taken among migrants.

This is likely to introduce no serious error. Since the number of deaths and births will be very small.

one combining ①, ② and ③ we get desired Result.

(ii) Logistic Curve ⇒

let the size of popⁿ at time t be P and at time $t+\Delta t$ be $P+\Delta P$. So rate of increase of popⁿ at time t is

$$\frac{dP}{dt} = \lim_{\Delta t \to 0} \frac{\Delta P}{\Delta t}$$

we may consider the relative growth rate of P i.e $\frac{1}{P}\frac{dP}{dt}$ and examine its behaviour as fⁿ of Time.

As the practical assumption would be that relative growth gradually decreases as t and P increases.

therefore. $\frac{1}{P}\frac{dP}{dt} = r(1-k \cdot P)$; $r, K > 0$

$$\Rightarrow \frac{d \log P}{dt} = r(1-k \cdot P) \Rightarrow \frac{1}{P(1-kP)} \cdot dP = r\,dt$$

$$\Rightarrow \left[\frac{1}{P} + \frac{K}{1-kP}\right] dP = r\,dt$$

$$\Rightarrow \log P - \log(1-kP) = rt + c$$

$$\Rightarrow \log\left(\frac{P}{1-kP}\right) = rt + c \Rightarrow \frac{P}{1-kP} = e^{rt} \cdot e^{c}$$

$$[A = e^{c}]$$

$$\Rightarrow P = (1-kP)A \cdot e^{rt}$$

$$\Rightarrow A = \frac{A e^{rt}}{1 + Ak e^{rt}} = \frac{1}{K + \frac{e^{-rt}}{A}} = \frac{1}{K\left[1 + \frac{e^{-rt}}{AK}\right]}$$

as $t \to -\infty$, $P \to 0$

$t \to \infty$, $P \to \frac{1}{K}$

let the upper limit of popⁿ be L i.e $\frac{1}{k} = L$

$$\Rightarrow \quad P = \frac{L}{1 + \frac{L e^{-rt}}{A}}$$

let at β time $P = \frac{L}{2}$ $\Rightarrow$ $\frac{L}{2} = \frac{L}{1 + \frac{L e^{-r\beta}}{A}}$

$$\Rightarrow \quad A = L e^{-r\beta}$$

$$\Rightarrow \quad P = \frac{L}{1 + e^{(\beta - t)r}}$$

Properties $\Rightarrow$

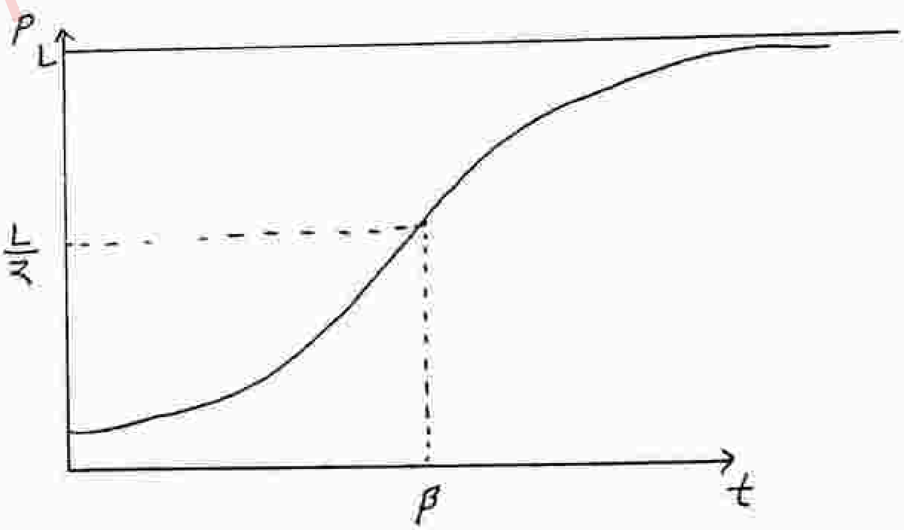
(1) $\frac{dP}{dt} > 0$ $\Rightarrow$ popⁿ increases with time t.

(2) $\frac{d^2P}{dt^2} = 0$ if $P = \frac{L}{2}$ i.e $t = \beta$

> 0 if $P < \frac{L}{2}$ i.e $t < \beta$ $\rightarrow$ Convex

< 0 if $P > \frac{L}{2}$ i.e $t > \beta$ $\rightarrow$ Concave.

and $P = 0$ as $t \to -\infty$ and $P = L$ as $t \to \infty$ are Two asymptotes.



Fitting of Logistic Curve ⇒

Method of Pearl and Reed ⇒

$$P_t = \frac{L}{1+e^{r(\beta-t)}}$$

let us suppose that the Logistic curve pass through three selected equidistant points $(0, P_0)$; (n, P_n), $(2n, P_{2n})$ so

$$\frac{1}{P_0} = \frac{1+e^{r\beta}}{L} \quad \text{---} \ *$$

$$\frac{1}{P_n} = \frac{1+e^{r(\beta-n)}}{L}$$

and $\dfrac{1}{P_{2n}} = \dfrac{1+e^{r(\beta-2n)}}{L}$

let $d_1 = \dfrac{1}{P_0} - \dfrac{1}{P_n}$ and $d_2 = \dfrac{1}{P_n} - \dfrac{1}{P_{2n}}$

$$\Rightarrow d_1 = \frac{e^{r\beta}(1-e^{-rn})}{L} \; ; \; d_2 = \frac{e^{r(\beta-n)}(1-e^{-rn})}{L}$$

$$\Rightarrow \frac{d_1}{d_2} = e^{rn} \Rightarrow r = \frac{1}{n}\log\left(\frac{d_1}{d_2}\right) \quad \text{---} (1)$$

Now $\dfrac{d_2}{d_1} = e^{-rn} \Rightarrow 1 - \dfrac{d_2}{d_1} = 1 - e^{-rn}$

$\Rightarrow \dfrac{d_1-d_2}{d_1} = \dfrac{Ld_1}{e^{r\beta}}$ from d_1.

$$\Rightarrow e^{r\beta} = \frac{Ld_1^2}{d_1-d_2} \Rightarrow \frac{e^{r\beta}}{L} = \frac{d_1^2}{d_1-d_2}$$

$\Rightarrow \left(\dfrac{1}{P_0} - \dfrac{1}{L}\right) = \dfrac{d_1^2}{d_1-d_2}$ by *

$$\Rightarrow \quad \frac{1}{P_0} - \frac{d_1^2}{d_1 - d_2} = \frac{1}{L} \quad — ②$$

r, L are obtained from ① and ②, for β, we use

$$* \quad \text{as} \quad e^{r\beta} = \frac{L}{P_0} - 1 \quad — ③$$

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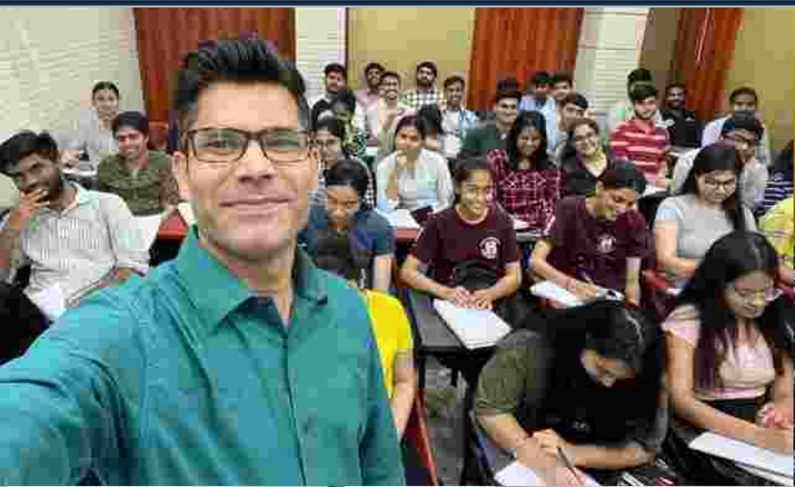

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